

Simulation of Geophysical Phenomena in the Laboratory

M. P. BACHYNSKI

RCA Victor Research Laboratories, Montreal, Canada

I. Introduction

THE exploration of "space" has become more and more an investigation of plasma-magnetic field interaction. This is because "space" consists primarily of plasma permeated by magnetic fields, and their mutual interactions give rise to the many geophysical phenomena observed on earth and more recently with space probes.

Since the earliest times, these observations and measurements have stimulated or have been stimulated by theoretical deduction. Consequently, a considerable effort, which is continuing to increase, has been expended in attempts to explain the various phenomena. Because of the complexity of the interaction in nature, a number of assumptions are necessary in each theoretical derivation. It is usually difficult to ascertain whether the physical mechanisms that are crucial to each theory are the important ones in nature, or whether effects that have been neglected really determine what happens.

More and more measurements, particularly those due to space probes, are beginning to produce significant data that can be used to resolve some of these aspects. One major difficulty is that the earth's environment is not a "controlled" environment, so that it is in many cases difficult to perform a true "controlled" experiment in space which may test a crucial feature of our understanding of a phenomenon, nor is a single measurement, no matter how precise, likely to be sufficient due to the cyclic (diurnal, sunspot, etc.) behavior of many natural phenomena. We are accumulating a considerable number of piecemeal observations but, as yet, no general unifying concept of the geophysical environment.

One can then ask, "could not scale model laboratory measurements provide considerable information on the applicability of various theories and consequently assist in the interpretation of full-scale data and perhaps suggest the design of future "space" experiments?" Over the last sixty

Table 1 Comparison of parameters found in nature and in laboratory

	Characteristic length, cm	Charge particle density, cm^{-3}	Particle velocity, cm/sec	Magnetic field, gauss
Galaxy	10^{22}	10^0	...	10^{-6}
Magnetosphere	10^{10}	10^1	10^7	10^{-3}
Ionosphere	10^9	10^3-10^6	...	10^{-1}
Laboratory	10^0-10^1	$10^{12}-10^{15}$	10^6-10^7	10^3-10^5

years, a limited number of significant scale model experiments have been conducted which have had a surprisingly large influence on the subsequent theoretical development. With the great strides presently being made in plasma technology, the number will undoubtedly continue to increase markedly in the near future.

This paper, after a brief examination of scaling considerations, summarizes the scale model experiments that have attempted to simulate, or are applicable to simulation of electromagnetic wave propagation in the ionosphere, aurora, solar wind-magnetosphere interaction, cosmic ray trajectories, and other geophysical phenomena.

II. Scaling Considerations

Laboratory scale model measurements can only be of value if the results of the laboratory experiment can be extrapolated in one form or another to the natural situation. One must, therefore, first consider the type of conditions that exist in nature and those that can be generated in the laboratory. An estimate of the characteristic lengths, charge particle density, particle velocity, and magnetic fields to be found in the galaxy, the magnetosphere, the ionosphere, and in the laboratory are tabulated in Table 1. The most significant feature to note is the wide spread (up to

Morrel P. Bachynski is Director of the Microwave and Plasma Physics Laboratories. He obtained his B.E. degree in Engineering Physics from the University of Saskatchewan in 1952, his M.S. degree the following year also from the University of Saskatchewan, and his Ph.D. in 1955 from McGill University. He has conducted research on electromagnetic wave propagation, microwaves, and various areas of plasma physics. Dr. Bachynski is a member of the American Physical Society and of the Canadian Association of Physicists, a Senior Member of the Institute of Electrical and Electronics Engineers, and an Associate Fellow of the Canadian Aeronautics and Space Institute. He is also Chairman of Commission VI of the Canadian National Committee of the International Scientific Radio Union (URSI).

22 orders of magnitude) in some of the parameters in going from nature into the laboratory. These figures will give some insight as to the magnitude of the ratios involved in the scaling.

The next general consideration is the parameters that should be simulated. The most important considerations include the following:

1) Ionization, dissociation, excitation and recombination potentials, lifetimes of excited and metastable states and various other inelastic atomic processes. It is at once obvious that these cannot be transformed and must remain the same in the natural as in the simulated laboratory scale.

2) Kinetics of the neutral and charged particles (electrons and ions), such as: a) particle trajectories, including drift and spiraling motion, b) particle energies and momenta including the directed and thermal components, and c) elastic collision processes, such as collision cross sections and mean-free paths.

3) Interrelation of the fields and field configurations.

4) Scale of a particular phenomena in size (characteristic length), time duration, and strength of the various interactions.

The laws of similitude for a gas discharge have been considered by Cobine,¹ based on the experimentally verified Paschens law whereby an increase in gas pressure does not alter the breakdown voltage of a given gas if all the dimensions are decreased correspondingly. If a is the scale factor that transforms a given system into, say, a laboratory system, then it is readily shown¹ that the mean-free path, length, and time transform as a^1 ; particle velocity (and hence particle energy), total current, and potentials transform as a^0 (i.e., they are unaltered); neutral particle density, pressure, and electric fields transform as a^{-1} ; and charge density and current density transform as a^{-2} . Most of the other significant parameters can be inferred from this discussion. For example, the conductivity scales as current density divided by electric field, i.e., as a^{-1} , etc. The field quantities are linked through Maxwell's equations and, since velocity (length/time) scales as a^0 , the magnetic field scales proportionately to the electric field, i.e., as a^{-1} . We thus see that, as first pointed out by Alfvén,² a linear scaling of a geophysical dimension to a laboratory scale by, say, a factor 10^{-8} requires an increase of the electric and magnetic fields of a factor 10^8 . This would require laboratory magnetic fields in the megagauss range. Thus, an exact simulation of nature, which satisfies all the laws of similitude, is clearly impossible. (In this discussion we have said nothing of the scaling of the various inelastic atomic processes that occur and that do not satisfy the aforementioned scaling laws. Even neglecting these considerations, similitude cannot be achieved.)

The question then arises as to whether any meaningful scaled experiments can be performed in the laboratory. The alternative to fulfilling the similitude conditions exactly is to scale the significant parameters only and to attempt to minimize the effects of the other factors, so that a particular aspect of the natural situation can be isolated and studied. The scaling considerations then involve considering the equations describing a particular interaction and determining the quantities to be simulated and those to be neglected. This implies that, in all probability, a different set of parameters, and hence a separate scale model experiment, is required in order to study each different phenomenon of nature. Such an approach was initially considered by Chapman³ for the case of simulating a magnetic storm and more recently for more general phenomena by Lehnert.⁴ In the subsequent sections, we shall consider the scaling considerations appropriate to the phenomena under discussion.

At best, only a distorted simulation of the important features can be achieved. Nevertheless, such laboratory experiments can serve an important purpose in guiding the

theoretical work. If a theory at least predicts the results of experiment in one range of parameters, one can have more confidence in the same theory over a different range of parameters. In the same manner, experimental results can serve to stimulate the direction of theoretical deduction. Finally, the techniques of measuring plasma phenomena in the laboratory are closely allied to the techniques used to determine plasma properties in nature so that scale model experiments in the laboratory can lead to the development of technology to be used in space probes for the exploration of natural plasmas.

III. Radio Wave Propagation in the Ionosphere

The scaling of the ionosphere to study radio wave interaction in the laboratory depends on relatively few parameters. These are the charged particle number density (n), the magnetic induction (B_0), the radio frequency of interest (ω), a frequency determining the average number of collisions a charged particle experiences (ν), and a characteristic length (L). If only electrons are considered in determining the electrical properties of the ionosphere, then the first two parameters are usually put in terms of the plasma frequency (ω_p) and the electron gyro or cyclotron frequency (ω_b), respectively, where $\omega_p^2 = (ne^2)/(m\epsilon_0)$; $\omega_b = (e/m)B_0$. e , m are the electronic charge and mass, respectively, and ϵ_0 is the permittivity of free space.

The characteristic parameters are, thus, in dimensionless form, ω_p/ω , ω_b/ω , ν/ω , and L/λ . Typical values for the ionosphere are: $\omega_p/\omega_b = 2-6$, $\nu/\omega_b = 10^{-4}-10^{-1}$, $\omega/\omega_b = 0.1-10$, $L/\lambda = 10^2-10^{-4}$.

These parameters can, in general, be simulated in the laboratory. The parameter presenting the greatest difficulty is a characteristic length, since the ionosphere is thousands of wavelengths in extent (effectively an infinite plasma), whereas a laboratory plasma is of necessity finite in extent. It is the effect of finite dimensions of the laboratory plasma which makes comparison with ionospheric experiments difficult. If the laboratory experiment can be designed so as not to be critically dependent on boundaries, then useful simulation measurements can be performed. Such experiments on different aspects of radio wave propagation in the ionosphere are summarized below.

A. Dielectric Coefficient of a Plasma

A controversy has existed for some time as to the form of the expression for the dielectric coefficient of a plasma which should be used in the equations of magnetoionic theory. The point in question is whether the motion of a representative "average" electron used to obtain an approximate description of the kinetics of a plasma in the presence of an electric field \mathbf{E} and magnetic induction \mathbf{B}_0 is governed by an equation of the form

$$m\ddot{\mathbf{x}} = e(\mathbf{E} + \dot{\mathbf{x}} \times \mathbf{B}_0) - m\nu\dot{\mathbf{x}} \quad (1)$$

(where \mathbf{x} is a linear dimension, and the dots denote time derivatives) or whether an extra "polarization" electric field due to the displacement of electrons in the neighboring molecules by the electric field also arises. This latter consideration, due to Lorentz,⁵ has met with some success in accounting for the refractive index of gases at great pressures. The equation of motion, taking into account this polarization term, takes the form

$$m\ddot{\mathbf{x}} = e(\mathbf{E} + (\mathbf{P}/3\epsilon_0) + \dot{\mathbf{x}} \times \mathbf{B}_0) - m\nu\dot{\mathbf{x}} \quad (2)$$

where the polarization $\mathbf{P} = nex$.

Many papers⁶⁻⁸ have been written as to which expression is the correct one and on the conditions under which either theory holds. A number of ionospheric experiments attempting to clarify this point are described by Ratcliffe.⁹ These measurements tend, in general, to substantiate Eq.

Table 2 Laboratory experiments on wave propagation in anisotropic plasmas

Authors	Magnetic field ω_b , gigacycles	θ , deg	Plasma frequency, gigacycles	Radio frequency, gigacycles	Remark
Goldstein ¹⁷ (1958)	to 70	0	10	$\pi - 2\pi \times 10$	Faraday rotation measurements in waveguides
Gallet et al. ¹⁸ (1960)	18-240	0	600	$2\pi \times 3.3$	Zeta machine, $\nu = 10^7 \text{ sec}^{-1}$
Consoli and Dagai ²⁰ (1961)	350	0	...	$2\pi \times 35$	Laboratory setup
Dellis and Weaver ²¹ (1962)	123	0	to 160	$2\pi \times 10$	Laboratory experiment
Mahaffey ²² (1963)	70	0	5-100	$2\pi \times 10$	Laboratory setup
Motley and Heald ¹⁹ (1960)	to 300	90	to 300	$2\pi \times 35$	B-1 stellarator
				$2\pi \times 70$	
Meservey and Schlesinger ²³ (1963)	to 500	0, 90	25-90	$2\pi \times 35$	C stellarator ($D = 34\lambda$)
				$2\pi \times 70$	
Dellis and Weaver ²⁴ (1964)	to 150	0	to 450	$2\pi \times 10$	Laboratory experiment

(1). Nevertheless, from these observations, the experimental evidence is not entirely conclusive, and there still exists some doubt on this point.

The same question has been asked in recent laboratory experiments,¹⁰⁻¹² and two quantitative laboratory measurements have been reported. Buchsbaum¹³ has observed the shift in resonant frequency of a cylindrical S-band cavity when loaded coaxially with a discharge in helium as function of the current through the plasma. His results agree well with these predicted by Eq. (1) and deviate significantly from the Lorentz predictions. Daiber and Glick¹⁴ arrive at a similar conclusion from measurements of the reflection of 3320 and 9375G cycle radio signals from plasmas generated in air by a shock tube. It thus appears that, for the types of plasmas commonly found both in the ionosphere and in the laboratory, the Lorentz polarization term should not be included in the expression for the dielectric coefficient of a plasma. One should note that the plasmas both in the ionosphere and in the laboratory experiments are of very low density (both electron number density and neutral background density). Theoretical considerations by Kadomtsev¹⁵ indicate that the Lorentz term provides a term of the order $1/n_D$, where n_D is the number of particles in a debye sphere. Hence, it is still conceivable that plasmas of high density and low temperature may yet demonstrate the Lorentz term type of dependence. Such plasmas are, however, of little interest in simulating ionospheric effects.

B. Wave Propagation in an Anisotropic Plasma

The Appleton-Hartree equation,¹⁶ which predicts the value of the refractive index μ of a cold unbounded plasma, has met considerable success in interpreting the interaction of radio waves with the ionosphere. The application of radio techniques for the measurement of plasma properties in the laboratory have implicitly assumed that the Appleton-Hartree equation was applicable, and the plasma properties were calculated from the measured parameters (phase and amplitude) on this basis.

Only recent plasma technology has advanced to the point where proper experiments can be designed which eliminate or at least minimize the practical limitations such as wall effects, refraction, reflection, and diffraction so that a test of the Appleton-Hartree equation is possible. Furthermore, plasmas of considerable extent, permeated by large magnetic fields, have become available through the effort devoted to thermonuclear plasma research. Hence, larger L/λ parameters, as discussed earlier, which simulate the ionosphere more closely, can be met.

The reported measurements¹⁷⁻²³ of $e-m$ wave propagation through anisotropic plasmas are tabulated in Table 2. In all the reported experimental work, the static magnetic field has been oriented either parallel to (interest in Whistler mode propagation) or perpendicular to the direction of wave propagation. No measurements exist for other than these

orientations. In particular, the results of Meservey and Schlesinger,²³ performed in the Stellarator C facility, afforded a plasma 34 free-space wavelengths in extent in a uniform magnetic field, which could be varied from 17 to 32.5 kgauss, thus providing a good environment for the testing of the Appleton-Hartree equation. In particular, their range of parameters permitted them to pass through the hybrid resonance condition for the extraordinary mode when the direction of propagation is normal to the direction of the static applied magnetic field. Their measurements agree very well with the predictions of the Appleton-Hartree equation. No experiments of either transmission or reflection of electromagnetic waves incident at an angle to a layer of plasma have been reported.

An interesting area in which laboratory measurement would be of considerable interest is the determination of radio wave propagation in plasmas where the electron collision frequency depends on electron energy. The effect of the energy variation of the electron collision frequency in the Appleton-Hartree equation has been worked out,²⁵⁻²⁷ and experimental verification of these theories would be of considerable importance.

C. Wave Interaction Phenomena

An electromagnetic wave traveling in an ionized region may, under certain circumstances, interact with a second wave in such a way that a modulation imposed on one of the waves becomes transferred to the other. This cross-modulation interaction phenomena in the ionosphere, first reported by Tellegen,²⁸ resulted from a modulation introduced by a high-power radio signal from Radio Luxembourg and since that time is often termed the "Luxembourg effect." This same type of phenomena can also occur for strong unmodulated radio wave interactions with their "self-effect," i.e., rf energy from the wave alters the conductivity of the plasma, which, in turn, affects the wave interaction.

A successful explanation of the cross-modulation effect was offered initially by Bailey and Martyn²⁹ and revised³⁰ to include the effect of the earth's magnetic field (anisotropic plasma). Recently renewed interest has developed in these effects in connection with rocket probing of the ionosphere, where a transmitter on board the rocket is used to modulate the ionosphere, while a second radio signal is used to sample the plasma (as well as in connection with satellites immersed in the ionosphere where the transmitters employed which, although of small power, may be sufficient to modify the surrounding plasma).

The cross-modulation and self-effects are well suited for laboratory study and were first observed in an isotropic plasma by Goldstein.^{31, 32} Using this technique, it was possible to determine the effective electron-neutral collision frequency and also to ascertain the electron-ion contribution to the conductivity.³³ Subsequent studies³⁴ led to a determination of the times for equipartition of energy between elec-

trons and ions due to Coulomb collisions. Further refinement enabled a study of electron-electron effects³⁵ in the plasma and electron temperature dependence of the recombination coefficient³⁶ by the same techniques. The cross-modulation and self-effects can also readily be generated in anisotropic plasma, and many of the aforementioned plasma kinetic effects have been studied.³⁷ With the development of these techniques to such a considerable degree of sophistication as a means of studying plasma in the laboratory,³⁸ it now may well be applied to the ionosphere to study some of the processes taking place there.

Other wave interaction effects that are considered to be of importance in ionosphere phenomena and that could lend themselves to laboratory simulation include the two-stream plasma instability,³⁹ Landau damping,⁴⁰ and heavy ion effects.⁴¹⁻⁴³

IV. Aurora

The earliest laboratory measurement aimed at simulating geophysical phenomena are those connected with aurora. These are the classic terrella experiments of Birkeland,⁴⁴ who, in order to verify his assertion that electrons traveling towards the earth would be deflected by the earth's magnetic field, performed a model experiment in which electrons were made to impinge on a small magnetized sphere in an evacuated chamber. These gas discharge experiments showed that the distribution of electrons near the sphere was determined principally by the magnetic field of the spherical terrella and in particular that luminous regions appeared around the magnetic poles of the terrella, indicating that many of the electrons were guided toward the polar regions. A middle belt surrounding the model equator and a toroidal space around it were free from charged particles. The results of these experiments played a major role in the thinking subsequently devoted to aurora and magnetic storm theory as developed by Birkeland and Störmer.⁴⁵ Although much of this theory has been superseded by subsequent discoveries, it formed a notable advance at the time and still remains a classic description of the trajectories of charged particles in a magnetic field. These ideas have met with considerable success in the theory of cosmic rays (see Sec. VI). Some of Birkeland's experiments were repeated and extended by Villard⁴⁶ and by Brüche.⁴⁷ Brüche's interest was in the influence of an equatorial ring current on the distribution of charged particles around the polar regions. He found that, by placing a ring around the equator of the model terrella and making electrical current flow in the ring, the middle belt free of charged particles could be extended and the polar zones made to move closer to each other. These results supposedly confirmed theoretical calculations made by Störmer.^{44, 45}

At the time of the Birkeland-Brüche experiments, both the physics of gas discharges and the dynamics of geophysical phenomena were very little understood. Hence, no attempt could really be made in scaling the significant parameters. Since a linear scaling of the fields is impossible because of the excessively large magnetic fields required in the laboratory, one is forced to try to scale the essential phenomena and ascertain how the remainder of the parameters are connected. The most important parameters become the electron and ion trajectories considering both the drift motion and the spiral motion in the combined magnetic field and space-charge electric potentials. In general, the drift motion of electrons can be simulated, but their relative spiral motion will be greater in the laboratory than in nature. If the radius of curvature of the electron spiraling motion $\rho = m(\mathbf{v} \times \mathbf{B}_0)/eB_0^2$ is less than R , the model earth size ($\rho < R$), then this limitation is not serious. On the other hand, the ion motion is exceedingly difficult to simulate. In all cases, the laboratory experiment must be performed at sufficiently low pressures so that collision effects are negligible. The de-

tailed scaling considerations must, of course, be worked out separately according to the theory that the model is being designed to simulate.

The most serious attempt at studying auroral phenomena by means of laboratory scale model experiments was inspired by the electric field theory of aurora and magnetic storms due to Alfvén.² Scale model experiments were initiated by Malmfors⁴⁸ in order to verify and extend some of the concepts of the electric field theory. In Malmfors' experiments, a magnetized terrella was suspended in a vacuum chamber between two insulated plates and a large potential applied between the plates. One of the plates had an aperture through which electrons from an electron gun could be injected. The terrella was coated with a fluorescent powder in order to be able to see the regions where electrons impinge upon the terrella. Under these conditions, two luminous rings formed surrounding the polar regions of the terrella. The magnetic field of the terrella was too weak to significantly deflect the electrons incident from the gun; as well, similar luminous regions could be observed in the absence of the electron beam but in the presence of a gas discharge (even if very weak). The process is thus due to secondary electrons created through ionization, whose energy is sufficiently low that they can be influenced by the magnetic field of the terrella and the electric field in the experimental configurations. The appearance and behavior of the luminous regions were interpreted to be in general accordance with the electric field theory of aurora.

The promising results of Malmfors were followed by more detailed experiments and a serious look at scaling considerations by Block.⁴⁹⁻⁵² In Block's experiments, the electron drift motions could be simulated, the spiral motion of the electrons, as expected, were relatively greater in the laboratory than in nature, and the ion motion could not be simulated. It was, however, possible to make collision effects negligible and to work with particle energies of the desired range.

A detailed series of measurements for different magnetic fields and plate potentials were performed by Block, extending the results obtained by Malmfors. As before, the general behavior of the simulated auroral zones, their position, and eccentricity could be interpreted within the framework of Alfvén's theory of the aurora.

No detailed attempts at laboratory scale model experiments to simulate other recent auroral theories have as yet been reported. It would, however, be interesting to compare such results with the investigations of the Alfvén group. [An experiment to simulate bundles of protons impinging on the earth (the Stormertron) as discussed by Bennett and Hulbert⁵³ has been reported. Since these phenomena seem to be more closely related to cosmic ray trajectories, these experiments are accordingly discussed in Sec. VI.] The only other simulation studies related to aurora have been published by Okada,⁵⁴ who reported some preliminary observations using the Stormertron on the influence of a radial electric field superimposed on a magnetic dipole field on the acceleration and precipitation of protons into the region of the polar cap.

V. Solar Wind-Magnetosphere Interaction

A large number of theories have evolved to explain magnetic storms and associated observations. Despite the various viewpoints, there appear some general areas of accord between the different theories. Most theories agree that these geophysical phenomena are caused by a "solar wind" impinging upon the magnetic field of the earth (which is approximately dipole in configuration), compressing the magnetic field on the windward side and slowing down the solar particles in the advancing stream. On the side of the earth away from the sun, the earth's magnetic field is distorted by the impinging solar stream, so that the mag-

Table 3 Comparison of solar wind-magnetosphere parameters⁶³

Parameter	Units	Values in space	Desired in laboratory	Obtained in laboratory
Ambient pressure of neutral density	mm Hg	10^{-18}	2×10^{-4} (max)	10^{-5}
Electron or ion density	cm^{-3}	5	10^{13} (min)	2×10^{13}
Streaming velocity	cm/sec	10^8	2×10^6 (min)	10^8
Electron temperature	°K	10^6	3×10^4	3×10^4
Uniform length of stream	cm	1.5×10^{13}	10 (min)	10
Radius of terrella	cm	6.37×10^8	3	3
Magnetic field at surface of terrella	gauss	0.311	2×10^3 (min)	5×10^3
Thickness of interface transition region	cm	5.3×10^4	0.93	1
Radius of cavity on sunlit side	cm	$5.14R_E = 3.28 \times 10^9$	$2.24R_E = 6.72$	$2R_E = 6$
Total time that wind acts	sec	10^6	10^{-2}	6×10^{-5}

netic field is confined to a hollow cavity. If the particles in the solar stream have thermal velocities, they will move so as to close the cavity, but it will be closed at a great distance from the earth on the side away from the wind. The net result is an elongated cavity (the magnetosphere) in which the magnetic field is confined.

All the theories also require a "ring current," a complex system of currents induced in the magnetosphere by the solar wind, which gives rise to the various phases of the magnetic storm and associated effects. The physical mechanism and detailed interaction of the solar wind-magnetosphere and the induced ring currents are, however, as varied as the number of proponents of the different theories (see, for example, Refs. 55-61).

The solar wind interacting with the magnetosphere can, for most considerations, be described as a fully ionized perfectly conducting plasma that behaves as a magnetohydrodynamic medium. This means that Ohm's law for such a conducting fluid moving with velocity \mathbf{V} in the presence of electromagnetic forces \mathbf{E} and \mathbf{B} can be written as

$$\mathbf{E} + (\mathbf{V} \times \mathbf{B}) = 0 \quad (3)$$

Replacing \mathbf{E} by its equivalent term from Maxwell's equations leads to

$$\partial \mathbf{B} / \partial t = \nabla \times (\mathbf{V} \times \mathbf{B}) \quad (4)$$

This equation implies that the magnetic flux through any loop moving with the local fluid velocity is constant in time, i.e., the lines of force are frozen into the fluid and are carried along with it.

In order to simulate the solar wind in the region of the magnetosphere as a fully ionized plasma, it is necessary that the effects of electron-neutral collisions be negligible. To consider whether it is possible to simulate an MHD region in the laboratory, it is necessary, as carefully done by Shkarofsky,⁶² to look at a more general form of Ohm's law for a plasma, which, neglecting ion slip effects, is

$$\mathbf{E} + (\mathbf{V} \times \mathbf{B}) = a \nabla p_e + b \mathbf{J} + c (\mathbf{J} \times \mathbf{B}) \quad (5)$$

where, now, electron pressure gradients, resistivity effects, and Hall forces affect the interaction. The coefficients a , b , c depend on electron and neutral particle densities, collision frequencies, and masses. Thus, in order to simulate an MHD medium in the laboratory,⁶² the a , b , c factors must be kept small. This implies

$$\frac{\kappa T}{e B_0 C_0 L} \ll 1 \quad \frac{m v_{ei}}{\mu_0 n e^2 C_0 L} \ll 1 \quad \frac{B}{\mu_0 n e C_0 L} \ll 1$$

where C_0 , L are characteristic velocities and lengths, respectively, of the experiment; m , n , e are the electron mass, number density, and charge, respectively; μ_0 is the permeability of free space, v_{ei} is the electron-ion collision frequency, and κ is Boltzmann's constant.

A further requirement of the solar wind is that the directed energy of the particles which comprise the wind be

much greater than the thermal energy of the particles. This implies $(2nMC_0^2)/(2n\kappa T) > 1$, where M is the mass of the ions.

In addition to insuring that all the preceding inequalities are satisfied (i.e., we have a MHD medium that behaves as a fully ionized gas in which the directed velocities of the ions exceed their thermal velocities), some thought must be given to the strength, size, and time duration of an interaction simulated in the laboratory.

In order that the solar wind interact strongly with the magnetic field (and be stopped by the field), the pressure exerted by the magnetic field must be of the same magnitude or slightly greater than the kinetic pressure of the solar wind, i.e., $B^2/2\mu_0 \gtrsim 2nMV^2$. It is desirable that the position of the balance point of magnetic and kinetic pressures (nearest approach of the wind) be located in approximately the same scale (in earth radii) from a model terrella as in the natural case. This implies approximate scaling of the cavity size. Considering the magnetic field to be of dipole configuration, the magnetic field B at the distance r is given in terms of the field B_e at the earth surface (distance r_e) by $r/r_e = (B_e/B)^{1/3}$.

Since the compressed field at the interface is considered to be increased by $(1.4)^4$ of the unperturbed value, then

$$\left(\frac{r}{r_e}\right) = \left(\frac{2.83 B_e}{B}\right)^{1/3} = \left(\frac{(2.83 B_e)^2}{4\mu_0 n M V^2}\right)^{1/6}$$

which is about 5-10 earth radii in the natural case. One thus would like the solar wind in the experiment to penetrate to 5-10 radii of the model earth.

A further requirement is that the cavity be sufficiently large (and the magnetic field sufficiently strong) so that the radius of curvature of the magnetic field lines be greater than the ion cyclotron radius in order that the spiraling motion of the charged particle trajectories be simulated, and hence "mirroring" effects in the boundary can occur. This is probably the most difficult condition to fulfill in the laboratory.

Finally, one would like to scale the time interaction of the solar wind in terms of the duration of its flow past the earth. In the natural situation, the sudden commencement of a magnetic storm lasts about 10^2 sec, the initial phase 10^4 sec, and the entire storm 10^5 sec. The time-scaling factor is basically limited by the timenecessary in order to perform the laboratory measurement. A value of 10^{-5} seems to be a reasonable value for the laboratory measurement, so that the time-scaling factor becomes 10^{-7} .

A comparison (due to Shkarofsky⁶³) of values of the various parameters in space and the desired values in the laboratory in order to satisfy the scaling conditions that have been discussed are shown in Table 3. In addition, the values of these parameters which can be obtained are also listed. In general, all the requirements except the total time during which the solar wind acts can be simulated with present techniques. In order to simulate the complete storm, a solar wind lasting

10^{-2} sec is required. (For a streaming velocity of 10^6 cm/sec, this implies a plasma of length 10^4 cm.) One can thus hope to study the initial commencement of the magnetic storm and perhaps some aspects of the initial phase. The remainder of the simulated magnetic storm, if it occurs, will take place in a vastly distorted and varying time scale.

A number of laboratory experiments on the interaction of a stream of plasma with magnetic dipoles of two- and three-dimensional configurations have been reported.⁶⁴⁻⁷⁵ These have been performed by propelling a stream of plasma with a plasma gun into a dipole magnetic field configuration. The majority of the observations are visual, showing the configuration of the plasma in the meridian plane containing the dipole axis and the plasma gun. The plasma has been shot at the equatorial plane of the magnetic field. A standoff region of plasma is observed on the windward side; the exact shape of the region is somewhat different for the various situations, undoubtedly due to the different conditions in the various experiments. These conditions are summarized in Table 4.

Osborne et al.^{70, 74, 75} and Cladis et al.,⁷² in addition to visual observations (Fig. 1), present measurements of the model earth magnetic field in the presence of the plasma. They observe the sweeping action of the plasma as it compresses the magnetic field in the equatorial plane on the windward side. The visual standoff region is observed at approximately the region where the plasma kinetic pressure equals the perturbed magnetic field pressure. On the leeward side, the magnetic field near the terrella shows a slight increase followed by a decrease in net value. The magnitude of the field increase diminishes rapidly with distance from the terrella.

A second visible discharge^{70, 74} has been seen surrounding the poles near the terrella surface and following along the magnetic field lines joining these regions (see Fig. 1). This visible region exhibits a well-defined minimum latitude

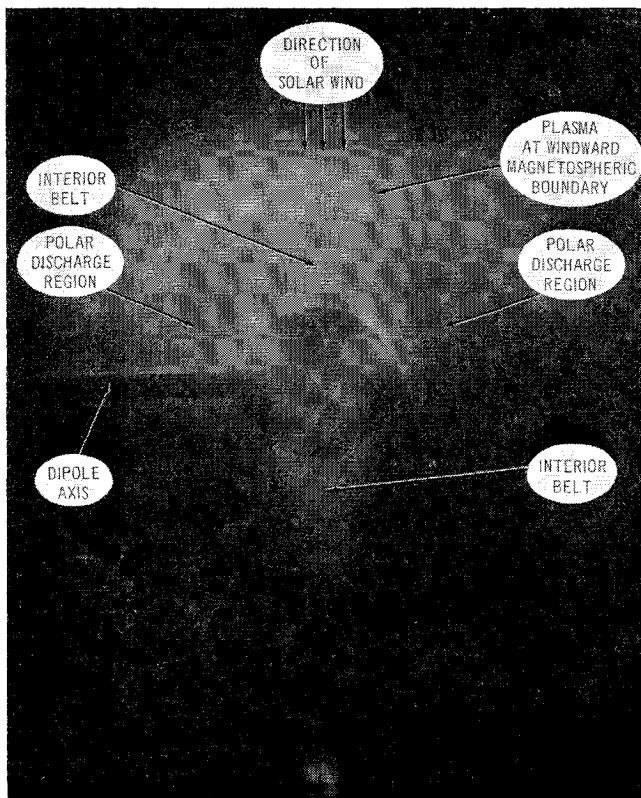


Fig. 1 Visual observation of plasma regions created in laboratory simulation experiments on interaction of solar wind and magnetosphere (after Osborne, Bachynski, and Gore⁷⁴).

Table 4 Experimental parameters for simulation of solar wind-magnetosphere interaction

Author	Solar Wind			Dipole Field			Type of measurement
	Pressure, torr	Density, cm^{-3}	Duration, μsec	Velocity, cm/sec	Configuration	Strength, in front of terrella	
Kawashima and Ishizuka ⁶⁷	$1-2 \times 10^{-3}$	10^{12}	5 μsec peak	10^7	3-dimensional dipole	1000 gauss 10 mm in front of terrella	6-mm-diam coil Visual
Kawashima ⁶⁸	10^{-5}	10^{14}	10 μsec	4×10^6	3-dimensional dipole	10,000 gauss	20-mm-diam terrella Visual, magnetic field
Bostick et al. ^{64, 65}	10^{-4}	10^{14}	10 μsec	2×10^6	2-dimensional dipole	2.5 kgauss 1 cm from coil	Rect. coil 5×37 cm Visual
Bostick et al. ⁶⁶	...	5×10^{13}	...	2×10^6	3-dimensional dipole	...	5 cm-diam loop coil Visual
Osborne, Shkarofsky, Gore ⁷⁰	10^{-6}	2×10^{13}	10 μsec peak	10^6	3-dimensional dipole	3 kgauss at equator	Terrella 6-cm diameter Visual, magnetic field
Alfvén et al. ⁷¹	$0-10^{-3}$	$10^{12}-10^{16}$	not specified	$2-5 \times 10^6$	3-dimensional dipole	10 kgauss at equator	Terrella 2.4-cm-diameter Visual, magnetic field
Cladis et al. ⁷²	2×10^{-6}	5×10^{12}	approx. 10 μsec	6×10^6	3-dimensional dipole	1 kgauss at equator to 10 kgauss at equator	Terrella 5.8 cm-diam permanent magnet Visual, magnetic field
Sellen and Bernstein ⁷³	10^{-6}	$1-5 \times 10^9$	300 μsec	$2-5 \times 10^6$	approx. 3 dimensional dipole	5000 gauss	C-electromagnetic Plasma potential, magnetic field
Osborne et al. ^{74, 75}	10^{-6}	2×10^{13}	60 μsec	2×10^6	3-dimensional dipole	4 kgauss at equator	Terrella 6-cm-diam Visual, magnetic field, elec. field

and is analogous in position to the outer Van Allen belt in space. This trapped region of plasma in the equatorial plane is larger on the windward side of the magnetic dipole than on the leeward side (consistent with particle motion on a constant L shell). The motion of the plasma in these visible regions is in a westward direction. Streak photographs indicate that this azimuthal velocity is about a third of the velocity of the impinging solar wind. Electrostatic and magnetic probe measurements show pronounced fluctuations in electric and magnetic fields corresponding to a form of magnetosonic or electrostatic wave within the trapped regions of plasma and peak electrical potential within the boundary.^{73, 74}

Further magnetic measurements^{70, 75} indicate a shift in the location of the "dip pole" under conditions of the magnetic field-plasma stream interaction. The dip pole is shifted from the dipole axis toward the windward side of the terrella, the angle between the "dip pole" axis and the dipole axis being a function of the "wind" parameters and the terrella field. This angle could be made as large as 20° . In addition, an isolated polar discharge is found along the direction of the "dip pole" lines. These observations are summarized in Fig. 2.

Two distinct types of solar wind-magnetosphere interactions depending on the properties of the solar wind have been identified.⁷⁵ The "standoff" interaction shows a complete standoff of the impinging solar wind plasma at several earth radii from the terrella. No visible injection of plasma material into the region of the terrella is observed. A very steep gradient of the magnetic field at the position of the standoff and the formation of discrete inner regions of visible plasma are found. The "injection" interaction is characterized by the visual injection at high latitudes of plasma into the vicinity of the terrella. Magnetic field measurements indicate a more gradual compression of the magnetic field and no discrete inner regions.

Such measurements show promise in possibly being able to separate some phenomena from others and hence guide theoretical speculation and will, no doubt, be pursued with greater vigor in future.

VI. Cosmic Ray Trajectories

With the discovery of cosmic radiation and the subsequent observation of latitude, diurnal, and east-west variations in the cosmic ray flux, the influence of magnetic fields on the trajectory of charged particles gained new importance in attempting to explain these effects.

The original calculations of Störmer,⁴⁵ although inspired by the solar stream theory of the aurora and developed before the discovery of cosmic radiation, are very well suited to determine the trajectories of cosmic ray particles in the environment of the earth's dipole magnetic field. Using this approach, it is possible to determine the momentum and trajectory of particles that will reach certain critical latitudes and also particles that can never reach the earth and that may be trapped to spiral around a bundle of magnetic lines of force. Such calculations become exceedingly involved,^{45, 76} even for a simple dipole magnetic field configuration, and become even more complicated if the effect of an electric field on the particle orbits is included.⁷⁷ It is thus not always possible to determine the particle orbits over as wide a range as desired because of the prohibitive length of the analytic computations.

The scaling of charged particle trajectories in a magnetic field is easier than the scaling of most other effects relating to geophysical phenomena. This is because of the limited number of parameters involved: the magnetic field (strength and direction) and the vector particle momentum. We can write the equation of motion of a charged particle as

$$d/dt(m\mathbf{v}) = (e/m)(m\mathbf{v} \times \mathbf{B}_0) \quad (6)$$

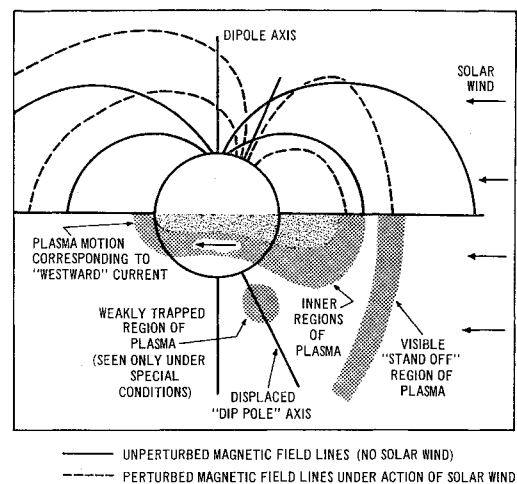


Fig. 2 Regions of plasma and distortion of magnetic field in presence of solar wind as found in laboratory simulation experiments.

If the unit of length is changed to the Störmer length where all distances are now measured in units of $(Me/mv)^{1/2}$, where M is the magnetic moment of three-dimensional dipole, then, as shown by Störmer,⁴⁵ the equation for the particle trajectories can be written in Cartesian coordinates as

$$\frac{d^2x}{ds^2} = \frac{3yz}{r^5} \frac{dz}{ds} - \frac{(3z^2 - r^2)}{r^5} \frac{dy}{ds} \quad (7a)$$

$$\frac{d^2y}{ds^2} = \frac{(3z^2 - r^2)}{r^5} \frac{dx}{ds} - \frac{3xz}{r^5} \frac{dz}{ds} \quad (7b)$$

$$\frac{d^2z}{ds^2} = \frac{3xz}{r^5} \frac{dy}{ds} - \frac{3yz}{r^5} \frac{dx}{ds} \quad (7c)$$

These are then universal equations for the particle trajectories which inherently contain the particle momentum and magnetic field strength for a dipole magnetic field. The scaling of the laboratory experiment then consists of choosing the parameters such that the model size consists of the same number of Störmer length units as the earth's parameters expressed in Störmer units. The energy of the incident particles to be used in the experiment is related to the momentum and hence to the Störmer unit. Therefore, the model parameters (momentum and magnetic field strength) are interrelated; having chosen one, the other is automatically determined. Scale model experiments to illustrate the charged particle orbits over varying conditions are, therefore, entirely feasible.

Two independent groups (Malmfors,⁷⁸ Brunberg and Dattner,^{79, 80} and Bennett^{81, 82}) have devised laboratory scale model experiments to demonstrate the variation of charged particle trajectories in the presence of magnetic fields. The approach of the Swedish group⁷⁸⁻⁸⁰ was that, since the trajectory of a negative particle leaving the earth is the same as that of a positive particle approaching the earth, a scale model experiment in which electrons were projected at fixed directions from a magnetized model earth would demonstrate the asymptotic direction of approach of observed cosmic ray particles that impinge on the earth at the position and direction of the model electron stream.

In the experiments, an electron beam was emitted in various directions from different latitudes on the surface of the model earth and its direction determined at a large distance from the magnetized model. By using such model experiments, they were able to demonstrate the deflection of charged particles of energies 10^9 – 10^{10} eV in the vicinity of the earth's magnetic field and hence ascertain the trajectory a charged particle undergoes in order that it be observed

incident from a given direction and at a specific point on the earth.

The threshold energy required by a particle, in order that it be capable of reaching a given region on the earth's surface, could be readily determined from the acceleration voltage accorded to the electron beam, and which, therefore, determined the electron energy.

Bennett⁸¹⁻⁸² developed a special laboratory tube that he called the Stormertron since it was designed to accurately produce the Stormer orbits to scale. Special attention was paid to shielding the high-energy electron beam that was projected towards a magnetized terrella from external electric fields. Many of the observations were made in low-density mercury vapor, which enabled the form of the beam to be readily seen and photographed. Fluorescent material on the magnetized model terrella demonstrated the position of impact of the particles reaching the earth's surface. In addition, the effect of a charged particle stream of significant diameter (which is difficult to calculate analytically) could be studied.

Both of these laboratory experiments could be used to confirm and extend the prediction of the theories of charged particle orbits in the presence of magnetic fields, and both served as a form of analog computer to demonstrate the more complicated effects.

VII. Other Phenomena

A theory of the formation of the solar system by Alfvén^{83, 84} includes an account of the mass distribution within the solar system including the planets and their satellites. An assumption fundamental to the theory is that, when a neutral gas is moving through a plasma such that the kinetic energy of a neutral particle has increased so as to be equal to its ionization energy, then the particle becomes ionized. Thus there exists a limiting velocity v_c for the neutral particle given by

$$Mv_c^2/2 = eV_{ion} \quad (8)$$

where M , V_{ion} are, respectively, the mass and ionization potential of the neutral particle. It is in this manner that a neutral gas falling toward a central body of mass M_c becomes ionized as it reaches the critical velocity v_c and consequently stopped by the magnetic fields that are assumed to prevail. Thus, stopping will occur when the gravitational energy equals the kinetic energy at the critical velocity, i.e., when

$$\gamma MM_c/R = Mv_c^2/2 \quad (9)$$

where γ is the gravitational constant. Equation (9) determines the distance R where matter will be accumulated. On this basis, it is possible to account for the positions of the planets and satellites. This assumption of the critical velocity depends on a rapid unexplained mechanism for ionization of the neutrals. (A theory by Lin⁸⁵ attributes the energy transfer to be caused by Coulomb collisions between the ions and the electrons, the electrons attaining sufficient energy to make ionizing collisions. In this way, a close coupling is provided between the kinetic energy of the ions and the ionization energy of the neutrals.)

Recent advances in plasma physics⁸⁶ have provided a means of experimentally studying in the laboratory the interaction between a neutral gas and a plasma in relative motion. Such an experiment has been carried out by Fahleson⁸⁷ and Angerth et al.⁸⁸ in a homopolar device in which the current is deliberately limited in order to avoid complete ionization of the gas. The experiment was carried out using a density of 10^{14} – 10^{16} atoms/cm³ in accordance with the required scaling factor between the astrophysical and laboratory situation. An electric discharge ionized the gas and the charged particles, acted upon by crossed electric and magnetic fields, were able to rotate around the central axis of the system. The nonionized components remained essentially at rest

and, hence, relative motion between the ionized gas and the neutral gas was achieved. In the experiment, the ionized component could be accelerated up to a certain critical velocity. The critical velocity for a partially ionized gas depends on the gas used, but is fairly independent of pressure, discharge current, and is proportional to the magnetic field. The critical velocity for a number of gases investigated (H, D, He, N, O, Ne, A) corresponds to that given in Eq. (8). More recent experiments by Wilcox et al.,⁸⁹ designed to simultaneously study ionization processes and the dynamics of fluid flow in magnetic fields, have been carried out in a linear geometry avoiding the complicating centrifugal forces of the homopolar devices. Again, a limiting velocity near to the critical velocity that is virtually independent of the pressure and discharge current is found. Other measurements⁹⁰⁻⁹² also show the existence of a limit on the rotating velocity which can be maintained without violent breakdown. Their limiting velocity does not correlate with the preceding theory, and loss of energy by ultraviolet radiation seems to be one of the dominant loss processes in the interaction. The velocity limit may, however, depend upon the neutral particle density in the plasma. It thus appears that the critical velocity limit does exist, but an accepted explanation requires yet further experimentation.

Recent laboratory techniques using neutralized ion beams^{93, 94} have been applied to the simulation of the interaction between a spacecraft such as a satellite in the ionosphere and its local environment. Such investigations give an indication of the plasma sheath formed on a satellite whose velocity is greater than the ion velocity and the influence of the sheath on local measurements and upon the electrical drag experienced by the satellite. Hall et al.⁹³ have made detailed measurements of the sheath potentials and ion current densities on and in the vicinity of a spherical model illuminated by a neutralized beam of cesium ions. They find complicated sheath distributions and ion flow patterns that depend upon the satellite potential. Shadow regions of ion current for a given satellite potential can become peaks of ion current for more negative vehicle potentials. The general features are in accord with the theory of Davis and Harris.⁹⁵ Knechtel and Pitts⁹⁴ have made direct measurements of electrical drag on a model satellite in the laboratory using a beam of mercury plasma. They conclude that the electrical drag can contribute significantly to the satellite ion drag and that the theory of Jastrow and Pearse,⁹⁶ which considers both the ion impact and the influence of the plasma sheath, agrees well with the measurements.

The preceding are good examples of the type of laboratory simulation experiments that can be performed in an effort to limit theoretical speculation. There are undoubtedly numerous other phenomena on which a great deal of information can be obtained from laboratory investigations. These include laboratory studies of Alfvén wave propagation as associated with geophysical phenomena, simulation of the earth-ionosphere sky wave propagation as a waveguide, etc., all of which can play an important part in our understanding of natural phenomena.

VIII. Conclusions

Plasma technology (both theory and experiment) has advanced to the position where it is becoming feasible to perform laboratory experiments designed to simulate various aspects of geophysical phenomena. Although an exact simulation of nature which satisfies all the laws of similitude is impossible, specific interactions can be studied by scaling the significant parameters and making the others negligible. This implies a separate scale model experiment in order to study each different phenomenon of nature. Such laboratory experiments can serve an important purpose in separating various phenomena from others and in this way limit, guide, and stimulate the theoretical work. In

some isolated instances, the laboratory measurements can serve as an analog computer for situations that are too complex to analyze in detail analytically. Furthermore, the techniques of measuring plasmas in the laboratory are clearly allied to the techniques used to determine plasma properties in nature, so that scale model experiments in the laboratory can lead to the development of technology to be used in space probes for the exploration of natural plasmas.

A number of laboratory experiments on the simulation of geophysical phenomena have been conducted throughout the years. These are summarized in the text. A great number of other geophysical phenomena could now be studied in limited form within the laboratory. There is little doubt that such experiments will become of increasing importance in the future.

References

- ¹ Cobine, J. D., *Gaseous Conductors* (Dover Publications, Inc., New York, 1958), p. 209.
- ² Alfvén, H., *Cosmical Electrodynamics* (Oxford University Press, London, 1948), p. 41.
- ³ Chapman, S., "Notes on aurora and magnetic storms—considerations as to a model experiment," *J. Atmospheric Terrest. Phys.* **1**, 189 (1951).
- ⁴ Lehnert, B., "Plasma physics on cosmical and laboratory scale," *Nuovo Cimento, Suppl.* **13**, 60 (1959).
- ⁵ Lorentz, H. A., *The Theory of Electrons* (B. G. Teuber, Leipzig, 1909).
- ⁶ Hartree, D. R., "The propagation of electromagnetic waves in a refracting medium in a magnetic field," *Proc. Cambridge Phil. Soc.* **27**, 143 (1931).
- ⁷ Darwin, C. G., "The refractive index of an ionized medium," *Proc. Roy. Soc. (London)* **A146**, 17 (1934); also "The refractive index of an ionized medium part II," *Proc. Roy. Soc. (London)* **A182**, 152 (1943).
- ⁸ Booker, H. G. and Berkner, L. V., "An ionospheric investigation concerning the Lorentz polarization term," *Terrest. Magnetic Atmospheric Electron.* **43**, 427 (1938).
- ⁹ Ratcliffe, J. A., *The Magneto-Ionic Theory and Its Application to the Ionosphere* (Cambridge University Press, London, 1959), Chap. 2.
- ¹⁰ Franklin, R. N. and Francis, G., "Dielectric constant of a plasma," *J. Electron. Control* **11**, 54 (1961).
- ¹¹ Gallup, J. W., Dutt, T. L., and Gibson, H., "Correspondence on 'dielectric constant of a plasma,'" *J. Electron. Control* **11**, 225 (1961).
- ¹² Nicoll, G. R. and Basu, J., "Comparison of microwave and langmuir probe measurements on a gaseous plasma," *J. Electron. Control* **12**, 23 (1962).
- ¹³ Buchsbaum, S. J., "Interaction of electromagnetic radiation with a high density plasma," Ph.D. Thesis, Dept. of Physics, Massachusetts Institute of Technology (1957); also Buchsbaum, S. J. and Brown, S. C., "The dielectric coefficient of a plasma," *J. Electron. Control* **12**, 573 (1962).
- ¹⁴ Daiber, J. W. and Glick, H. S., "Plasma studies in a shock tube," *Proceedings of the Symposium on Electromagnetics and Fluid Dynamics of Gaseous Plasma* (Polytechnic Press, Brooklyn, N. Y., 1962), p. 323.
- ¹⁵ Kadomtsev, B. B., "On the effective fields in a plasma," *Soviet Phys.* **6**, 117 (1958).
- ¹⁶ Appleton, E. V., "Wireless studies of the ionosphere," *J. Inst. Elec. Engrs.* **71**, 642 (1932).
- ¹⁷ Goldstein, L., "Nonreciprocal electromagnetic wave propagation in ionized media," *Trans. Inst. Radio Engrs.* **MITT-6**, 19 (January 1958).
- ¹⁸ Gallet, R. M., Richardson, J. M., Wieder, B., Ward, G. B., and Harding, G. H., "Microwave whistler mode propagation in a dense laboratory plasma," *Phys. Rev. Letters* **4**, 347 (April 1960).
- ¹⁹ Motley, R. and Heald, M. A., "Use of multiple polarization for electron density profile measurements in high-temperature plasmas," *Proceedings of the Symposium on Millimetre Waves* (Polytechnic Press, Brooklyn, N. Y., 1960), p. 141.
- ²⁰ Consoli, T. and Dagai, M., "Polarisation rotatoire magnetique dans les plasmas—application de la densite electronique," *J. Nucl. Energy Pt. C3*, 115–128 (1961); also "Measure de la densite electronique par effect faraday," *Proceedings of the Fifth International Conference on Ionization Phenomena in Gases* (North-Holland Publishing Co., Amsterdam, 1962), p. 423; also "Application de la propagation des ondes droites polarisées circulairement au diagnostic U.H.F.," *Proceedings of the Fifth International Conference on Ionization Phenomena in Gases* (North-Holland Publishing Co., Amsterdam, 1962), p. 465.
- ²¹ Dellis, A. N. and Weaver, J. M., "Measurement of a high refractive index for transmission of 3 cm microwaves through a plasma," *Nature* **193**, 1274 (1962).
- ²² Mahaffey, D. W., "Microwave propagation through a plasma in a magnetic field," *Phys. Rev.* **129**, 1481 (1963).
- ²³ Meservey, E. B. and Schlesinger, S. P., "Extraordinary mode dispersion for millimeter microwaves near the electron cyclotron frequency," *Matterhorn TR MATT-Q-20*, p. 57 (December 1962); also *American Physical Society Paper T-11* (June 1963).
- ²⁴ Dellis, A. N. and Weaver, J. M., "Whistler mode propagation in a laboratory plasma," *Proc. Phys. Soc.* **83**, 473–489 (1964).
- ²⁵ Jancel, R. and Kahan, T., "Theorie du couplage des ondes electromagnetiques ordinaire et extraordinaire dans un plasma inhomogene et anisotrope et conditions de reflexion, applications a l'ionosphere," *J. Phys. Radium* **16**, 136 (1955); also "Theorie non Maxwellienne des plasmas homogenes et anisotropes," *Nuovo Cimento* **12**, 573 (1954).
- ²⁶ Sen, H. K. and Wyller, A. A., "On the generalization of the Appleton-Hartree magneto-ionic formulae," *J. Geophys. Res.* **65**, 3931 (1960).
- ²⁷ Shkarofsky, I. P., "Values of the transport coefficients in a plasma for any degree of ionization based on a Maxwellian distribution," *Can. J. Phys.* **39**, 1619 (1961); also "Generalized Appleton-Hartree equation for any degree of ionization and application to the ionosphere," *Proc. Inst. Radio Engrs.* **49**, 1857 (1961).
- ²⁸ Tellegen, B. D. H., "Interaction between radio waves?," *Nature* **131**, 840 (1933).
- ²⁹ Bailey, V. A. and Martyn, D. F., "The influence of electric waves on the ionosphere," *Phil. Mag.* **18**, 369 (1934).
- ³⁰ Bailey, V. A., "On some effects caused in the ionosphere by electric waves," *Phil. Mag.* **26**, 425 (1938).
- ³¹ Goldstein, L., Anderson, J. M., and Clark, G. L., "Interaction of microwaves propagating through a gaseous discharge," *Phys. Rev.* **90**, 131 (1953).
- ³² Goldstein, L., "Electrical discharge in gases and modern electronics," *Advances in Electronics and Electron Physics* (Academic Press, New York, 1955), Vol. 7, p. 399.
- ³³ Anderson, J. M. and Goldstein, L., "Interaction of electromagnetic waves of radiofrequency in isothermal plasmas: collision cross-section of helium atoms and ions for electrons," *Phys. Rev.* **100**, 1037 (1955).
- ³⁴ Dougal, A. A. and Goldstein, L., "Energy exchange between electron and ion gases through coulomb collision in plasmas," *Phys. Rev.* **109**, 615 (1958).
- ³⁵ Goldstein, L. and Sekiguchi, T., "Electron-electron interaction and heat conduction in gaseous plasma," *Phys. Rev.* **109**, 625 (1958).
- ³⁶ Chen, C. L., Leiby, C. C., and Goldstein, L., "Electron temperature dependence of the recombination coefficient in pure helium," *Phys. Rev.* **121**, 1391 (1960).
- ³⁷ Goldstein, L., Narasinga Rao, K. V., and Verdyen, J. T., "Interaction of microwaves in gaseous plasmas immersed in magnetic fields," *Proceedings of the Symposium Electromagnetic and Fluid Dynamics of Gaseous Plasma* (Polytechnic Press, Brooklyn, N. Y., 1962), p. 121.
- ³⁸ Kaliszewski, T., "Laboratory observations of the Luxembourg effect in free space plasma columns," *J. Electron. Control* **XIV**, 81 (1963).
- ³⁹ Farley, D. T., Jr., "Two-stream plasma instability as a source of irregularities in the ionosphere," *Phys. Rev. Letters* **10**, 279 (1963).
- ⁴⁰ Scarf, F. L., "Landau damping and the attenuation of whistlers," *Phys. Fluids* **5**, 6 (1962).
- ⁴¹ Hines, C. O., "Heavy ion effects in audio frequency radio propagation," *J. Atmospheric Terrest. Phys.* **11**, 36 (1957).
- ⁴² Buchsbaum, S. J., "Resonance in a plasma with two ion species," *Phys. Fluids* **3**, 418 (1960).
- ⁴³ Buchsbaum, S. J., "Ion resonance in a multicomponent plasma," *Phys. Rev. Letters* **5**, 495 (1960).
- ⁴⁴ Birkeland, K., *The Norwegian Aurora Polaris Expedition of 1905-03* (Christiania, Oslo, 1908, 1913); also Chapman, S. and Bartels, J., *Geomagnetism* (Oxford University Press, London, 1940), Vol. II, p. 831.

- ⁴⁵ Störmer, G., *The Polar Aurora* (Oxford University Press, 1955); also "Sur les trajectoires des corpuscules electriques dans l'espace sous l'action du magnetisme terrestre avec application aux aurores boréales," *Arch. Sci. Physiol.* (Geneve) **24**, 5, 113, 221, 317 (1907); **32**, 163 (1911); **35**, 483 (1913); also "Resultats des calculs numeriques des trajectoires des corpuscules electriques dans le champ d'un aimant élémentaire," *Kristiania, Skifter Videnskapsselsk. Mat.-Naturv. Pl. Nr. 1* (1913), **2** (1913), **3** (1913), **4** (1936), **5** (1936); also "Periodische Electronenbahnen im Feld eines Elementarmagneten und ihre Anwendung auf Brüches Modellversuche und auf Eschenhagens Elementarwellen des Erd Magnetismus," *Z. Astrophys.* **1**, 237 (1930).
- ⁴⁶ Villard, M. P., *Ann. Chem. Phys.* **9**, (1906); also *Radium* (Paris) **4**, (1907).
- ⁴⁷ Brüche, E., "Some new theoretical and experimental results on aurorae," *Terrest. Mag.* **36**, 41 (1931); also *Z. Astrophys.* **2**, 30 (1931).
- ⁴⁸ Malmfors, K. G., "Experiments on the aurorae," *Arkiv. Math. Astrofys.* **348**, 1 (1947).
- ⁴⁹ Block, L., "Model experiments on aurorae and magnetic storms," *Tellus* **7**, 65 (1955).
- ⁵⁰ Block, L., "On the scale of auroral model experiments," *Tellus* **8**, 234 (1956).
- ⁵¹ Block, L., "The present state of the electric field theory of magnetic storms and aurorae," *International Astronautical Union Symposium*, edited by B. Lehnert (Cambridge University Press, London, 1958), p. 312.
- ⁵² Block, L., "On the shape of the auroral zones," *Arkiv. Fysik* **14**, 153 (1958).
- ⁵³ Bennett, W. H. and Hulbert, E. O., "Magnetic self-focussed solar ion streams as the cause of aurora," *J. Atmospheric Terrest. Phys.* **5**, 211 (1954); also "Magnetic self-focussing of auroral protons," *Phys. Rev.* **91**, 1562 (1953).
- ⁵⁴ Okada, J. M., "Laboratory experiments concerning a polar cap injection mechanism," *Arkiv. Geofysik* **3**, 473-478 (1961).
- ⁵⁵ Chapman, S. and Ferraro, V. C. A., "A new theory of magnetic storms," *Terrest. Magnetic Atmospheric Electron.* **36**, 77, 171 (1931); **37**, 147, 421 (1932); **38**, 79 (1933); also "The theory of the first phase of a geomagnetic pulse," *Terrest. Magnetic Atmospheric Electron.* **45**, 245 (1940).
- ⁵⁶ Alfvén, H., "A theory of magnetic storms and of the aurora," *Kungl. Svenska Vetenskapsakad. Handlingar* **18**, no. 3, 1 (1939); **18**, no. 9, 3 (1940); also "On the electric field theory of magnetic storms and aurora," *Tellus* **VII**, 50 (1955).
- ⁵⁷ Parker, E. N., "Dynamics of the interplanetary gas and magnetic fields," *Astrophys. J.* **128**, 664 (1955); also "The hydrodynamic theory of solar corpuscular radiation and stellar winds," *Astrophys. J.* **132**, 821 (1960); also "Interaction of the solar wind with the geomagnetic field," *Phys. Fluids* **1**, 171 (1958).
- ⁵⁸ Gold, T., "Motions in the magnetosphere of the earth," *J. Geophys. Res.* **64**, 1219 (1959).
- ⁵⁹ Piddington, J. H., "Geomagnetic storm theory," *J. Geophys. Res.* **65**, 93 (1960).
- ⁶⁰ Axford, W. I. and Hines, C. O., "A unifying theory of high latitude geophysical phenomena and geomagnetic storms," *Can. J. Phys.* **39**, 1433 (1961).
- ⁶¹ Dungey, J. W., "Interactions of solar plasma with the geomagnetic field," *Planetary Space Sci.* **10**, 233 (1963).
- ⁶² Shkarofsky, I. P., "The solar wind, the magnetosphere and their simulation in the laboratory," *RCA Victor Research Rept.* 7-801-22 (October 1962).
- ⁶³ Shkarofsky, I. P., "Laboratory simulation of the solar wind-magnetosphere interaction," *Royal Society of Canada Meeting* (1963).
- ⁶⁴ Bostick, W. H., Byfield, H., Nakagawa, Y., and Wells, D., "Interaction between plasma and a two-dimensional magnetic dipole," *Phys. Fluids* **5**, 1305 (1962).
- ⁶⁵ Bostick, W. H., Byfield, H., and Brettschneider, M., "Measurement of B, E and n inside the simulated 'geomagnetic cavity in the presence of solar wind,'" *Phys. Fluids* **6**, 1361 (1963).
- ⁶⁶ Bostick, W. H., Byfield, H., Brettschneider, M., "Plasma flow around a three-dimensional dipole," *J. Geophys. Res.* **68**, 5315 (1963).
- ⁶⁷ Kawashima, N. and Ishizuka, H., "Interaction of a plasma stream with a three-dimensional magnetic dipole," *J. Phys. Soc. (Japan)* **18**, 736 (1963).
- ⁶⁸ Kawashima, N., "Interaction of a plasma stream with a three-dimensional dipole II," *J. Phys. Soc. (Japan)* **19**, 227 (1964).
- ⁶⁹ Osborne, F. J. F., "Experiments on simulation of the solar wind-magnetosphere interaction," *Royal Society of Canada Meeting* (1963).
- ⁷⁰ Osborne, F. J. F., Shkarofsky, I. P., and Gore, J. V., "Laboratory simulation of the solar wind-magnetosphere interaction," *Can. J. Phys.* **41**, 1747 (1963).
- ⁷¹ Alfvén, H., Danielsson, L., Fälthammer, C. G., and Lindberg, L., "On the penetration of interplanetary plasma into the magnetosphere," *Symposium on Plasma Space Science* (1963).
- ⁷² Cladis, J. B., Miller, T. D., and Baskett, J. R., "Interaction of a supersonic plasma stream with a dipole magnetic field," *J. Geophys. Res.* **69**, 2257-2272 (1964).
- ⁷³ Sellen, J. M., Jr. and Bernstein, W., "Interaction of collisionless plasma streams with transverse magnetic fields," *Phys. Fluids* **7**, 977-981 (1964).
- ⁷⁴ Osborne, F. J. F., Bachynski, M. P., and Gore, J. V., "Laboratory measurements of plasma trapping within the magnetosphere," *Appl. Phys. Letters* **5**, 77 (August 15, 1964).
- ⁷⁵ Osborne, F. J. F., Bachynski, M. P., and Gore, J. V., "Laboratory studies of the variation of the magnetosphere with solar wind properties," *J. Geophys. Res.* **69**, (1964).
- ⁷⁶ Lemaître, G. and Vallarta, M. S., "On the geomagnetic analyses of cosmic radiation," *Phys. Rev.* **49**, 719 (1936).
- ⁷⁷ Block, L., "Cosmic ray orbits in interplanetary magnetic fields," *Arkiv. Fysik* **14**, 161 (1958).
- ⁷⁸ Malmfors, K. G., "Determination of orbits in the field of a magnetic dipole with application to the diurnal variation of cosmic radiation," *Arkiv. Math. Astron. Fysik* **324**, 1 (1945).
- ⁷⁹ Brunberg, E. A. and Dattner, A., "Experimental determination of electron orbits in the field of a magnetic dipole," *Tellus* **5**, 2 (1953).
- ⁸⁰ Brunberg, E. A., "Cosmic rays in the terrestrial magnetic dipole field," *Tellus* **8**, 215 (1955).
- ⁸¹ Bennett, W. H., "Stormer orbits," *Proc. Symp. Appl. Math.* **IX**, 19 (1959).
- ⁸² Bennett, W. H., "Solar proton stream forms with a laboratory model," *Rev. Sci. Instr.* **30**, 63 (1959).
- ⁸³ Alfvén, H., *On the Origin of the Solar System* (Oxford University Press, London, 1954); also "Collision between a non-ionized gas and a magnetized plasma," *Rev. Mod. Phys.* **32**, 710 (1960); also "On the mass distribution in the solar system," *Astrophys. J.* **136**, 1005 (1962).
- ⁸⁴ Alfvén, H. and Wilcox, J. M., "On the origin of the satellites and the planets," *Astrophys. J.* **136**, 1016 (1962).
- ⁸⁵ Lin, S. C., "Limiting velocity for a rotating plasma," *Phys. Fluids* **4**, 1277 (1961).
- ⁸⁶ Wilcox, J. M., "Review of high temperature rotating plasma experiments," *Rev. Mod. Phys.* **31**, 1045 (1959).
- ⁸⁷ Fahleson, U. V., "Experiments with plasma moving through neutral gas," *Phys. Fluids* **4**, 123 (1961).
- ⁸⁸ Angerth, B., Block, L., Fahleson, U., and Soop, K., "Experiments with partially ionized rotating plasmas," *Nucl. Fusion, Suppl. Pt. I*, 39 (1962).
- ⁸⁹ Wilcox, J. M., Pugh, E., Dattner, A., and Eninger, J., "Experimental study of the propagation of an ionizing wave in a coaxial plasma gun," *Proceedings of the Case Institute Symposium on Plasma Guns* (to be published).
- ⁹⁰ Baker, D. A., Hammel, J. E., and Ribe, F. L., "Rotating plasma experiments I—Hydromagnetic properties," *Phys. Fluids* **4**, 1534 (1961).
- ⁹¹ Baker, D. A. and Hammond, J. E., "Rotating plasma experiments II—Energy measurements and the velocity limiting effect," *Phys. Fluids* **4**, 1549 (1961).
- ⁹² Halbach, K., Baker, W. R., and Layman, R. W., "Production of a hot rotating plasma," *Phys. Fluids* **5**, 1482 (1962).
- ⁹³ Hall, D. F., Kemp, R. F., and Sellen, J. M., Jr., "Plasma-vehicle interaction in a plasma stream," *AIAA J.* **2**, 1032-1039 (1964).
- ⁹⁴ Knechtal, E. D. and Pitts, W. C., "Experimental investigation of electric drag on satellites," *AIAA J.* **2**, 1148-1151 (1964).
- ⁹⁵ Davis, A. H. and Harris, I., "Interaction of a charged satellite with the ionosphere," *Rarefied Gas Dynamics*, edited by L. Talbot (Academic Press, New York 1961), pp. 691-699.
- ⁹⁶ Jastrow, R. and Pearce, C. A., "Atmospheric drag on the satellite," *J. Geophys. Res.* **62**, 413-423 (1957).